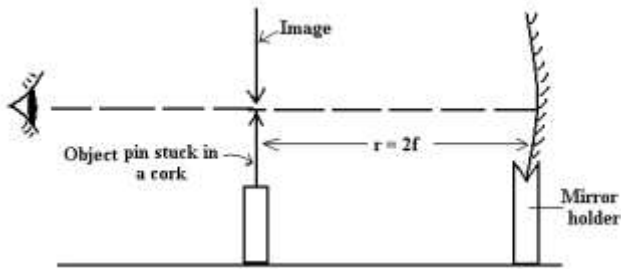


## PHYSICS LIGHT NOTES CONTINUATION

### 4.0 Determination of the Focal Length of a Concave Mirror.

#### 4.0.1 Method (1) Using a pin at C



An object pin is placed in front of a mounted concave mirror so that its tip lies along the axis of the mirror.

The position of the pin is adjusted until it coincides with its image such that there is no parallax between the pin and its image. The distance  $r$  of the pin from the mirror is measured.

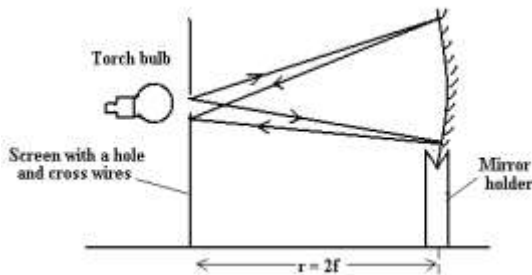
The required focal length

NOTE :

(i) In the position where there is no parallax between the object pin and its image, there is no relative motion between the object and its image when the observer moves the head from side to side.

(ii) When the pin coincides with its image, the rays are incident normal to the mirror and are thus reflected along their own path. Therefore the pin coincides with its image at the Centre of curvature of the mirror.

#### 4.0.2 Method (2) Using an illuminated object at C

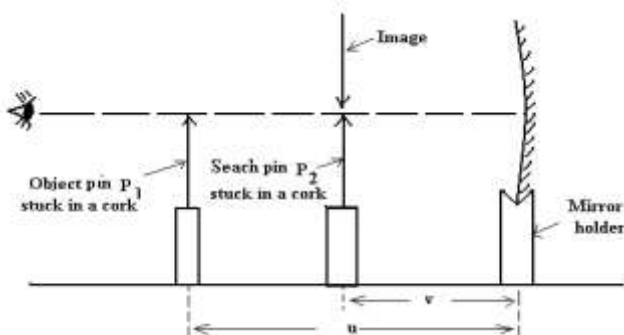


An illuminated bulb, a screen and a concave mirror mounted in a holder are aligned as shown above. The mirror position is adjusted to or from the screen until a sharp image of the cross-wire is formed on the screen besides the object.

The distance  $r$  of the mirror from the screen is measured.

The required focal length

#### 4.0.3 Method (3) Using no parallax method in locating $v$



An object pin  $P_1$  is placed at a distance  $u$  in front of a mounted concave mirror so that its tip lies along the axis of the mirror.

A search pin  $P_2$  placed between the mirror and pin  $P_1$  is adjusted until it coincides with the image of pin  $P_1$  by no-parallax method.

The distance  $v$  of pin  $P_2$  from the mirror is measured.

The procedure is repeated for several values of  $u$  and the results are tabulated including values of  $uv$ , and  $u+v$ .

A graph of  $uv$  against  $u+v$  is plotted and the slope  $s$  of such a graph is equal to the focal length  $f$  of the mirror.

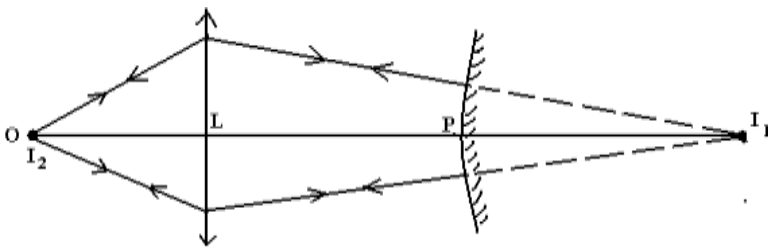
NOTE:

If a graph of  $\frac{1}{u}$  against  $\frac{1}{v}$  is plotted, then each intercept  $c$  of such a graph is equal

to  $\frac{1}{f}$ . **Thus  $f = \frac{1}{c}$**

## 1.1 Determination of Focal Length of a Convex Mirror.

### 4.1.1 Method (1) Using a convex lens.



An object  $O$  is placed in front of a convex lens to form a real image on the screen at  $I_1$ .

The distance  $LI_1$  of the screen from the lens is then measured.

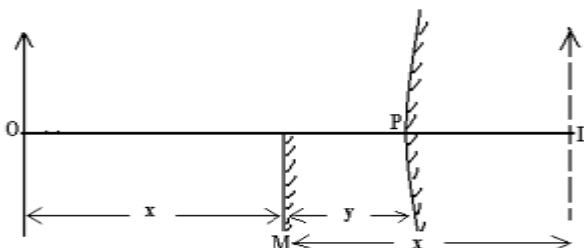
A convex mirror is placed between the lens and the screen and the mirror is then moved along the axis  $OI_1$  until an image  $I_2$  is formed besides  $O$ .

The distance  $LP$  is measured. The required focal length  $f = \frac{PI_1}{2}$ , where  $PI_1 = LI_1 - LP$

NOTE ;

When the incident rays from an object are reflected back along the incident path, a real inverted image is formed besides the object in which case the rays strike the mirror normally. Therefore they will if produced pass through the Centre of curvature of the mirror thus distance  $PI_1 = \text{radius of curvature}$

### 4.1.2 Method (2) Using No parallax.



An object pin  $O$  is placed in front of a convex mirror and a virtual diminished image is formed at  $I$ .

A plane mirror **M** is placed between **O** and **P** so as to intercept half the field of view of the convex mirror.

Mirror **M** is adjusted until its own image of **O** coincides with **I** by no parallax method. Measure the distances **x** and **y**.

The focal length of the mirror is the calculated from

**Note:**

- (i) The two images coincide when they are as far behind the plane mirror as the object is in front.
- (ii) Substituting for  $u = x + y$  and  $v = y - x$  in the mirror formula gives

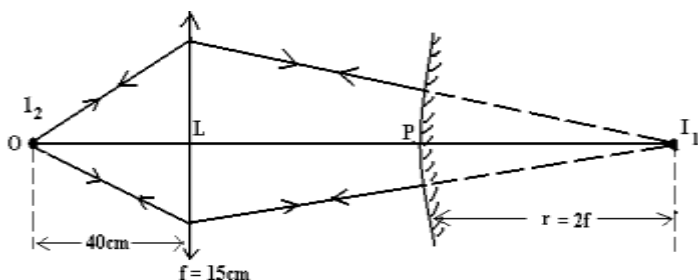
#### EXAMPLES

1. An object **O** is placed **40cm** in front of a convex lens of focal length **15cm** forming an image on the screen. A convex mirror situated **4cm** from the lens in the region between the lens and the screen forms the final image besides object **O**.

(i) Draw a ray diagram to show how the final image is formed.

(ii) Determine the focal length of the convex mirror.

**Solution**



Consider the action of a convex lens

$u = 40\text{cm}$ , and  $f = 15\text{cm}$ , Using the lens formula  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$  gives

$$v = \frac{f \cdot u}{u - f} = \frac{15 \times 40}{40 - 15} = 24\text{cm}, \text{ The radius of curvature } r = (24 - 4) \text{ cm} = 20\text{cm}$$

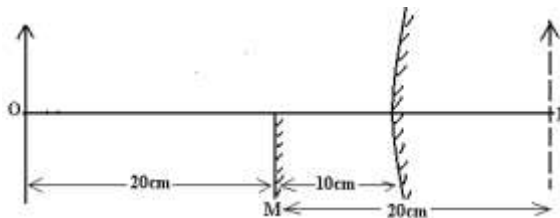
Using the relation  $r = 2f$

$$\Rightarrow 2f = 20\text{cm}$$

$$\therefore f = 10\text{cm}$$

Thus  $f = -10\text{cm}$  "The Centre of curvature of a convex mirror is virtual"

4. A plane mirror is placed **10cm** in front of a convex mirror so that it covers about half of the mirror surface. A pin **20cm** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.



Consider the action of a convex mirror

$u = 30\text{cm}$  and  $v = -(20 - 10) = -10\text{cm}$  "The image formed is virtual"

Using the mirror formula gives,

### EXERCISE

1. Describe an experiment to determine the focal length of a concave mirror.
2. You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a concave mirror, and a meter ruler. Describe an experiment to determine the focal length of a concave mirror using the above apparatus.
3. Describe an experiment, including a graphical analysis of the results to determine the focal length of a concave mirror using a no parallax method.
4. Describe an experiment to measure the focal length of a convex mirror
5. Describe how the focal length of a diverging mirror can be determined using a convex lens.
6. Describe how the focal length of a convex mirror can be obtained using a plane mirror and the no parallax method.
7. A plane mirror is placed at a distance **d** in front of a convex mirror of focal length **f** such that it covers about half of the mirror surface. A pin placed at a distance **L** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. With the aid of an illustration, Show that  $2df = d^2 - L^2$

## 5.0 REFRACTION OF LIGHT

**Refraction** is the bending of light rays at an interface between two media of different optical densities

NOTE ;

Light is refracted because it has different speeds in different media.

### 5.0.1 Laws of Refraction

Consider a ray of light incident on an interface between two media as shown.



O = Point of incidence.

AO = Incident ray

OB = Refracted ray.

ON = Normal at O

$\angle i$  = Angle of incidence

$\angle r$  = Angle of refraction

#### LAW 1:

The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.

#### LAW 2:

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media of different optical densities

Thus

#### REFRACTIVE INDEX. [ $n$ ]

is the ratio of the sine of angle of incident to the sine of angle of refraction for a ray of light traveling from air in to a given medium.

OR

Is the ratio of the speed of light in a vacuum to speed of light in a medium.

Thus **Refractive index,  $n = \frac{\text{speed of light in a vacuum, } c}{\text{speed of light in a medium, } c_m}$**

Where speed of light in a vacuum  $c = 3.0 \times 10^8 \text{ ms}^{-1}$ .

NOTE:

The refractive index, **n** for a vacuum is **1**. However if light travels from air to another medium, the value of **n** is slightly greater than **1**. For example, **n = 1.33** for water and **n = 1.5** for glass.

## THE PRINCIPLE OF REVERSIBILITY OF LIGHT.

It states that the paths of light rays are reversible. This means that a ray of light can travel from medium **1** to **2** and from **2** to **1** along the same path.

## RELATIONS BETWEEN REFRACTIVE INDICES

### CASE I:

Consider a ray of light traveling from **medium 1 (air)** to **medium 2 (glass)** as shown.



Suppose **i<sub>1</sub>** and **i<sub>2</sub>** are the angles of incidence and refraction respectively in **medium 1** and **medium 2**, then

For light traveling from **(1)** to **(2)**, refractive index  ${}_1n_2 = \frac{\sin i_1}{\sin i_2}$  -----(i)

For light traveling from **(2)** to **(1)**, refractive index  ${}_2n_1 = \frac{\sin i_2}{\sin i_1}$

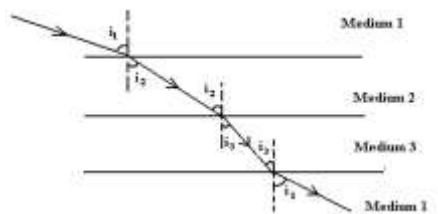
$$\Rightarrow \frac{1}{{}_2n_1} = \frac{\sin i_1}{\sin i_2} \text{ -----(ii) (reciprocal of the above equation)}$$

Equating equation (i) and (ii) gives

$${}_1n_2 = \frac{1}{{}_2n_1} \quad \text{OR} \quad {}_1n_2 \times {}_2n_1 = 1$$

### CASE II:

Consider a ray of light moving from **medium 1 (air)** through a series of media **2,3** and then finally emerge into **medium 1 (air)** as shown.



At **medium 2-medium 3** interface, Snell's law gives.

$${}_2n_3 = \frac{\sin i_2}{\sin i_3} = \frac{\sin i_2}{\sin i_1} \times \frac{\sin i_1}{\sin i_3} = {}_2n_1 \times {}_1n_3$$

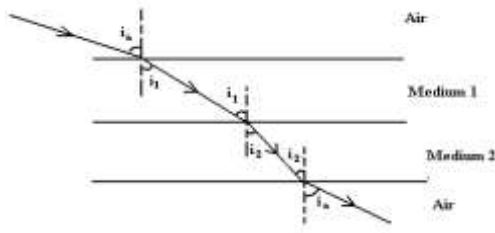
$$\therefore {}_2n_3 = {}_2n_1 \times {}_1n_3$$

Using the relation  ${}_2n_1 = \frac{1}{{}_1n_2}$

$$\Rightarrow {}_1n_3 = {}_1n_2 \times {}_2n_3 \quad \text{OR} \quad {}_2n_3 = \frac{{}_1n_3}{{}_1n_2}$$

## GENERAL RELATION BETWEEN $n$ AND $\sin i$

Consider a ray of light moving from air through a series of media 1, 2 and then finally emerge into air as shown.



At air – medium 1 interface, Snell's gives  $\frac{\sin i_a}{\sin i_1} = n_1$

$$\Rightarrow \sin i_a = n_1 \sin i_1 \text{ ..... (i)}$$

At air - medium 2 interface, Snell's gives  $\frac{\sin i_a}{\sin i_2} = n_2$

$$\Rightarrow \sin i_a = n_2 \sin i_2 \text{ .....(ii)}$$

Equating equation (i) and (ii) gives

$$n_1 \sin i_1 = n_2 \sin i_2.$$

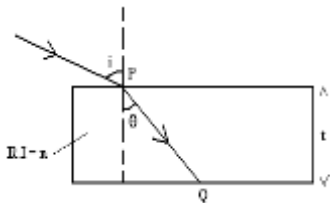
$\therefore n \sin i = \text{a constant.}$

### EXAMPLES:

1. Monochromatic light incident on a block of material placed in a vacuum is refracted through an angle  $\theta$ . If the block has a refractive index  $n$  and is of thickness  $t$ , show that this light takes a time  $\frac{n t \sec \theta}{c}$  to emerge from the block.

where  $c$  is the speed of light in a vacuum.

Solution:



Let  $T$  be the time taken by light to travel from point P to Q in the medium.

$$\text{Thus } T = \frac{\text{distance PQ}}{\text{speed of light in the block, } c_m}.$$

$$\text{Where distance PQ} = \frac{t}{\cos \theta} = t \sec \theta$$

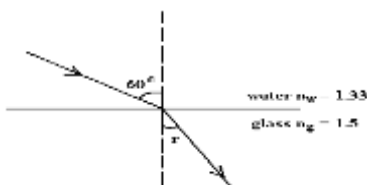
$$\Rightarrow T = \frac{t \sec \theta}{c_m} \text{ .....(i)}$$

$$\text{By definition, } n = \frac{c}{c_m} \Rightarrow c_m = \frac{c}{n} \text{ .....(ii)}$$

Substituting equation (ii) in to (i) gives

$$T = \frac{n t \sec \theta}{c}$$

2. A monochromatic beam of light is incident at  $60^\circ$  on a water-glass interface of refractive index 1.33 and 1.5 respectively as shown



Calculate the angle of reflection  $r$ .

**Solution:**

Applying Snell's law gives

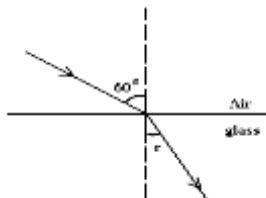
$$n_w \sin 60^\circ = n_g \sin r$$

$$1.33 \sin 60^\circ = 1.5 \sin r$$

Thus  $\angle r = 50.2^\circ$ .

3. A ray of light propagating from air is incident on an air-glass interface at an angle of  $60^\circ$ . If the refractive index of glass is **1.5**, calculate the resulting angle of refraction.

Solution.



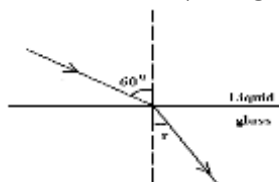
Applying Snell's law gives

$$n_a \sin 60^\circ = n_g \sin r \quad \text{but } n_a = 1, \quad n_g = 1.5$$

$$\Rightarrow 1 \sin 60^\circ = 1.5 \sin r$$

$$\therefore \angle r = 35.6^\circ.$$

4. A monochromatic ray of light is incident from a liquid on to the upper surface of a transparent glass block as shown.



Given that the speed of light in the liquid and glass is  $2.4 \times 10^8 \text{ ms}^{-1}$  and  $1.92 \times 10^8 \text{ ms}^{-1}$  respectively, find the angle of refraction,  $r$ .

Solution:

Applying Snell's law gives

$$n_l \sin 60^\circ = n_g \sin r$$

$$\frac{c}{c_l} \sin 60^\circ = \frac{c}{c_g} \sin r$$

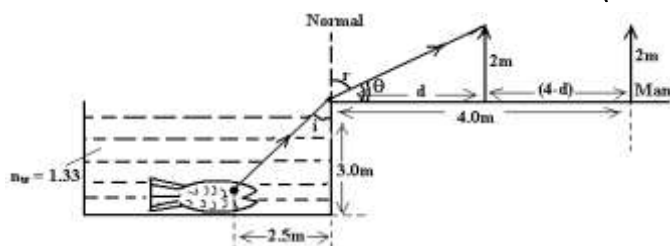
$$c_l \sin r = \frac{c_g}{c_l} \sin 60^\circ$$

$$\Rightarrow \sin r = \frac{c_g}{c_l} \sin 60^\circ \quad \text{but } c_g = 1.92 \times 10^8 \text{ ms}^{-1}, \quad c_l = 2.4 \times 10^8 \text{ ms}^{-1}$$

$$\therefore \sin r = \frac{1.92 \times 10^8}{2.4 \times 10^8} \sin 60^\circ$$

$$\Rightarrow \angle r = 43.9^\circ.$$

5. A small fish is 3.0m below the surface of the pond and 2.5m from the bank. A man 2.0m tall stands 4.0m from the pond. Assuming that the sides of the pond are vertical, calculate the distance the man should move towards the edge of the pond before movement becomes visible to the fish. (**Refractive index of water = 1.33**).



From the diagram,  $\tan i = \frac{2.5}{3} \Rightarrow \angle i = 39.81^\circ$

Applying Snell's law at the edge of the pond gives

$$n_w \sin i = n_a \sin r$$

$$1.33 \sin 39.81^\circ = 1 \sin r$$

$$\Rightarrow \angle r = 58.4^\circ$$

$$\text{Thus } \angle \theta = 90^\circ - 58.4^\circ = 31.6^\circ$$

$$\text{From the diagram } \tan \theta = \frac{2}{d} \Rightarrow d = \frac{2}{\tan 31.6^\circ}$$

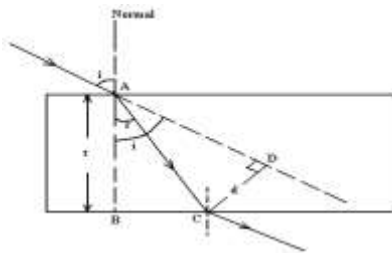
$$\therefore d = 3.2\text{m.}$$

$$\begin{aligned} \text{Thus required distance traveled} &= 4 - d \\ &= 4 - 3.2 \\ &= 0.8\text{m} \end{aligned}$$

### SIDE WISE DISPLACEMENT OF LIGHT RAYS.

When light travels from one medium to another, its direction is displaced side ways. This is called lateral displacement.

Consider a ray of light incident at an angle  $i$  on the upper surface of a glass block of thickness  $t$ , and then suddenly refracted through an angle  $r$  causing it to suffer a sideways displacement  $d$ .



$$\text{From } \triangle ABC, \quad AC = \frac{t}{\cos r} \quad \text{.....(i)}$$

$$\text{From the diagram, } \angle CAD = (i - r)$$

$$\text{From } \triangle ACD, \quad AC = \frac{d}{\sin (i - r)} \quad \text{.....(ii)}$$

Equating equation (i) and (ii) gives

$$\frac{t}{\cos r} = \frac{d}{\sin (i - r)}$$

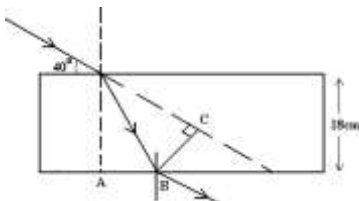
$$\Rightarrow d = \frac{t \sin (i - r)}{\cos r}$$

NOTE :

The horizontal displacement of the incident ray ,  $BC = t \cdot \tan r$

**Example:**

The figure below shows a monochromatic ray of light incident from a liquid of refractive index 1.33 onto the upper surface of a glass block of refractive index 1.42

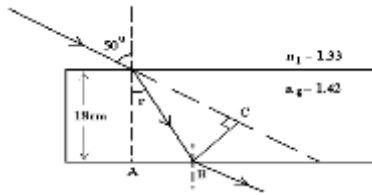


Calculate the;



- (i) horizontal displacement AB.  
(ii) lateral displacement BC of the emergent light.

**Solution**



- (i) Applying Snell's law at the liquid- glass interface gives,

$$n_l \sin 50^\circ = n_g \sin r.$$

$$1.33 \sin 50^\circ = 1.42 \sin r$$

$$\Rightarrow \angle r = 45.8^\circ$$

$$\begin{aligned} \text{Horizontal displacement AB} &= t \tan r \\ &= 18 \tan 45.8^\circ \\ &= 18.51 \text{ cm} \end{aligned}$$

(ii) Lateral displacement  $d = \frac{t \sin (i-r)}{\cos r}$

$$d = \frac{18 \sin (50^\circ - 45.8^\circ)}{\cos 45.8^\circ}.$$

$$d = 1.89 \text{ cm}.$$

## EXERCISE 1

(1) What is meant by **refraction of light**?

(2) (i) State the laws of refraction of light.

(ii) State what brings about refraction of light as it travels from one medium to another.

(3) (i) What is meant by the **refractive index** of a material?

(ii) Light of two colours blue and red is incident at an angle  $\gamma$  from air to a glass block of thickness  $t$ . When blue and red lights are refracted through angles of  $\theta_b$  and  $\theta_r$  respectively, their corresponding speeds in the glass block are  $v_b$  and  $v_r$ . Show that the separation of the two colours at the bottom of the glass block

$$d = \frac{t}{c} \left[ \frac{v_r}{\cos \theta_r} - \frac{v_b}{\cos \theta_b} \right] \sin \gamma \quad \text{Where } \theta_r > \theta_b \text{ and } c \text{ is the speed of light in air.}$$

(iii) Light consisting of blue and red is incident at an angle of  $60^\circ$  from air to a glass block of thickness **18 cm**. If the speeds of blue and red light in the glass block are  $1.86 \times 10^8 \text{ ms}^{-1}$  and  $1.92 \times 10^8 \text{ ms}^{-1}$  respectively, find the separation of the two colours at the bottom of the glass block..

[Answer: 0.54 cm]

(4) Show that when the ray of light passes through different media separated by plane boundaries,  $n \sin \phi = \text{constant}$  where  $n$  is the absolute refractive index of a medium and  $\phi$  is the angle made by the ray with the normal in the medium.

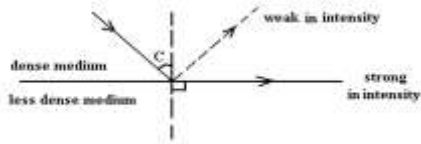
(5) Show that when the ray of light passes through different media **1** and **2** separated by plane boundaries,  ${}_1n_2 \times {}_2n_1 = 1$  where  $n$  is the refractive index of a medium.

(6). Show that when the ray of light passes through different media **1,2** and separated by plane boundaries,  ${}_1n_3 = {}_1n_2 \times {}_2n_3$  where  $n$  is the refractive index of a medium.

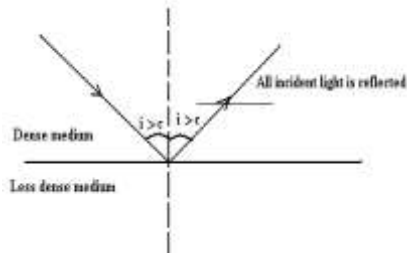
(7) Show that a ray of light passing through a glass block with parallel sides of thickness  $t$  suffers a sideways displacement  $d = \frac{t \sin(\phi - \lambda)}{\cos \lambda}$  where  $\phi$  is the angle of incidence and  $\lambda$  is the angle of refraction.

# CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Consider a ray of light incident on an interface between two media as shown.



**critical angle:** is the angle of incidence in the dense medium which results into an angle of refraction of  $90^\circ$  in the adjoining less dense medium. If the angle of incidence is greater than the critical angle, all the incident light energy is reflected back in the dense medium and **total internal reflection** is said to have occurred.



NOTE :

(a) At critical point Snell's law becomes.

$$n_1 \sin c = n_2 \sin 90^\circ$$

$$\Rightarrow \sin c = \frac{n_2}{n_1}$$

$$\therefore c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

Where  $n_1$  and  $n_2$  are the refractive indices of the dense and the less dense medium respectively. However if the less dense medium is air, then Snell's law becomes

$$n_1 \sin c = n_a \sin 90^\circ$$

$$\Rightarrow \sin c = \frac{1}{n_1}$$

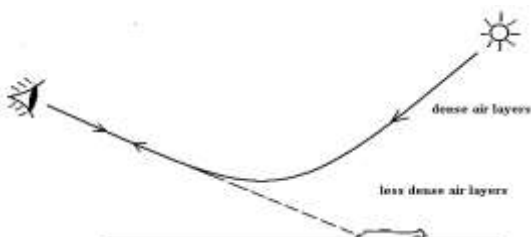
(b) The conditions for total internal reflection to occur are;

- (i) Light should travel from an optically denser to a less dense medium.
- (ii) The angle of incidence at the boundary of the media should be greater than the critical angle.

## APPLICATION OF TOTAL INTERNAL REFLECTION.

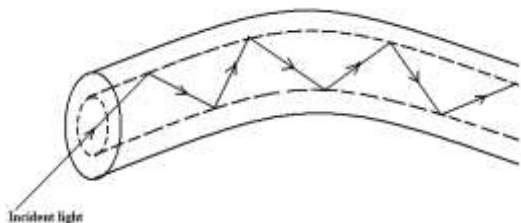
- (i) It is responsible for the formation of a mirage.
- (ii) It is responsible for the formation of a rainbow.
- (iii) It is responsible for the transmission of light in optical fibres.
- (iv). It is responsible for the transmission of sky radio waves
- (v). It is responsible for the transmission of light in prism binoculars.

## FORMATION OF A MIRAGE



On a hot day, The air layers near the earth's surface are hot and are less dense than the air layers above the earth's surface. Therefore as light from the sky pass through the various layers of air, light rays are continually refracted away from the normal till some point where light is totally internally reflected. An observer on earth receiving the totally internally reflected light gets an impression of a pool of water on the ground and this is the virtual image of the sky.

## AN OPTICAL FIBRE

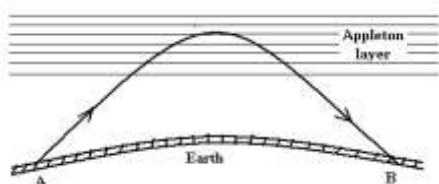


When a light ray enters in to the fibre, it bounces from one edge to another by total internal reflection. These successive total internal reflections enable the transmission of light in optical fibres.

### NOTE

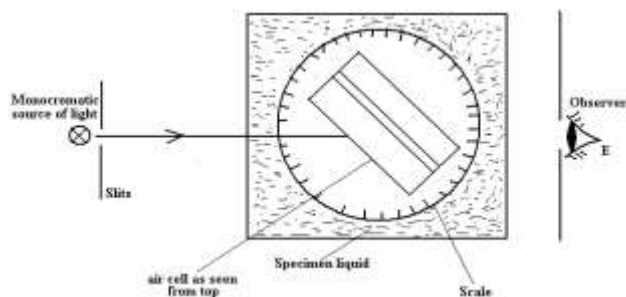
- (i) An optical fibre finds a practical application in an endoscope, a device used by doctors to inside the human body.
- (ii) Optical fibres are used in telecommunication systems (i.e. Telephone or TV signals are carried along optical fibers by laser light).

## SKY RADIO WAVES



A radio wave sent skyward from a station transmitter **A** is continually refracted away from the normal on entering the electron layer that exists above the earth's surface. Within the electron layer, the wave is totally internally reflected causing it to emerge from the electron layers and finally returns to the earth's surface where it's presence can be detected by a radio receiver at **B**

## DETERMINATION OF REFRACTIVE INDEX OF A LIQUID BY AN AIR-CELL METHOD

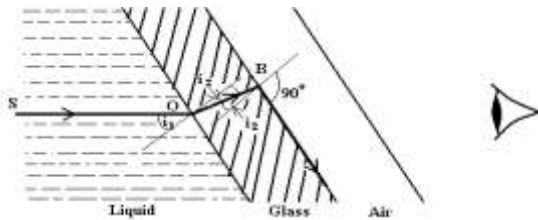


The air cell is immersed in a liquid under test. A beam of monochromatic light is directed onto the air cell and then observed through the cell from the opposite side at **E**. The cell is then rotated on one side until light is suddenly cut off and the angular position  $\theta_1$  is noted. The cell is again rotated in the opposite direction until light is suddenly cut off and the angular position  $\theta_2$  is noted. The refractive index of the liquid can then be calculated from  $n = \frac{1}{\sin \theta}$  Where  $\theta = \frac{\theta_1 + \theta_2}{2}$

NB:

The source of light should be monochromatic so that the extinction of light is sharp since monochromatic light does not undergo dispersion, as it is with white light.

## THEORY OF THE AIR-CELL METHOD



Ray  $OS$  is refracted along  $OB$  in glass. However, at  $B$  total internal just begins.

Suppose  $i_1$  is the angle of incidence in the liquid,  $i_2$  is the angle of incidence in the glass while  $n$  and  $n_g$  are the corresponding refractive indices, Then applying the relation  $n \sin i = \text{a constant}$  gives

$$n \sin i_1 = n_g \sin i_2 = n_a \sin 90^\circ$$

$$\therefore n \sin i_1 = n_a \sin 90^\circ$$

$$\Rightarrow n \sin i_1 = 1$$

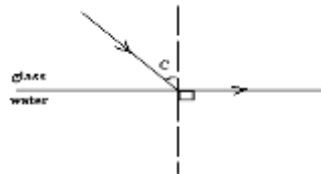
$$\text{Thus } n = \frac{1}{\sin i_1}$$

$$\text{But } \angle i_1 = \frac{\theta_1 + \theta_2}{2} \quad \text{Hence } n = \frac{1}{\sin \theta}$$

### Examples:

1 The critical angle for water-air interface is  $48^\circ 42'$  and that of glass-air interface is  $38^\circ 47'$ . Calculate the critical angle for glass-air interface.

Solution



Applying Snell's law gives  $n_g \sin c = n_w \sin 90^\circ$ ----- (i).

Given that for water-air interface  $c_w = 48^\circ 42'$ .

$$\therefore n_w \sin c_w = 1$$

$$n_w \sin 48^\circ 42' = 1$$

$$\Rightarrow n_w = 1.33$$
----- (ii)

Also for glass-air interface  $c_g = 38^\circ 47'$

$$n_g \sin c_g = 1$$

$$\therefore n_g \sin 38^\circ 47' = 1$$

$$\Rightarrow n_g = 1.67$$
----- (iii)

Substituting equation (ii) and (iii) into equation (i) gives

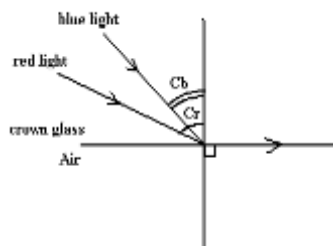
$$1.67 \sin c = 1.33 \sin 90^\circ$$

$$\Rightarrow c = 52.8^\circ$$

2. The refractive index for red light is 1.634 of crown glass and the difference between the critical angles of red and blue light at the glass-air interface is  $0^\circ 56'$ . What is the refractive index of crown glass for blue light

Solution

Analysis the critical angle between two media for red light is greater than that for any other light colour. This gives rise to the ray diagram below



Since  $c_r > c_b$ , then  $c_r - c_b = 0^\circ 56'$  -----(i)

Applying Snell's law to red light gives

$$n_r \sin c_r = 1$$

$$1.63 \sin c_r = 1$$

$$\Rightarrow c_r = 37.73^\circ$$

Equation (i) now becomes

$$c_b = c_r - 0^\circ 56'$$

$$c_b = 37.73^\circ - 0^\circ 56'$$

$$c_b = 36.8^\circ$$

Applying Snell's law to blue light gives

$$n_b \sin c_b = 1$$

$$n_b \sin 36.8^\circ = 1$$

$$\Rightarrow n_b = 1.67$$

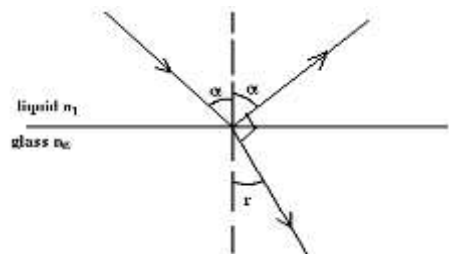
2. (i) A glass block of refractive index  $n_g$  is immersed in a liquid of refractive index  $n_l$ . A ray of light is partially reflected and refracted at the interface such that the angle between the reflected and the refracted ray is  $90^\circ$ . Show that

$n_g = n_l \tan \alpha$  where  $\alpha$  is the angle of incidence from the liquid to glass.

(ii) When the procedures in (i) above are repeated with the liquid removed, the angle of incidence increases by  $8^\circ$ .

Given that  $n_l = 1.33$ , find  $n_g$  and the angle of incidence at the liquid-glass interface.

Solution



Applying Snell's law at the liquid-glass interface gives

$$n_g \sin r = n_l \sin \alpha$$

$$\text{But } r + 90^\circ + \alpha = 180^\circ$$

$$\Rightarrow r = 90^\circ - \alpha$$

$$\therefore n_g \sin (90^\circ - \alpha) = n_l \sin \alpha.$$

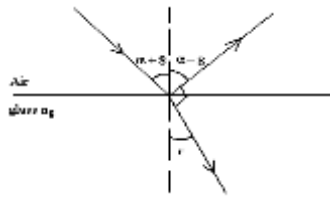
$$\text{From trigonometry, } \sin (90^\circ - \alpha) = \cos \alpha$$

$$\Rightarrow n_g \cos \alpha = n_l \sin \alpha.$$

Dividing  $\cos \alpha$  on both sides gives

$$n_g = n_l \tan \alpha.$$

(ii) When the liquid is removed.



$$\text{From } n_g = n_l \tan \alpha \text{-----(i)}$$

$$\Rightarrow n_g = n_a \tan (\alpha + 8^\circ) \quad \text{but } n_a = 1$$

$$\therefore n_g = \frac{\tan \alpha + \tan 8^\circ}{1 - \tan \alpha \tan 8^\circ}$$

$$n_g (1 - \tan \alpha \tan 8^\circ) = \tan \alpha + \tan 8^\circ$$

$$n_g - n_g \tan \alpha \tan 8^\circ = \tan \alpha + \tan 8^\circ \text{-----(ii)}$$

$$\text{from equation (i) } \tan \alpha = \frac{n_g}{n_l}$$

Substituting for  $\tan \alpha$  in equation (ii) gives

$$n_g - \frac{n_g^2 \tan 8^\circ}{n_l} = \frac{n_g}{n_l} + \tan 8^\circ$$

$$\text{but } n_l = 1.33.$$

$$\therefore n_g^2 - 2.340 n_g + 1.326 = 0 \text{-----(iii)}$$

Equation (iii) is quadratic in  $n_g$  and solving it gives

$$n_g = 1.39 \text{ or } n_g \text{ not physically possible.}$$

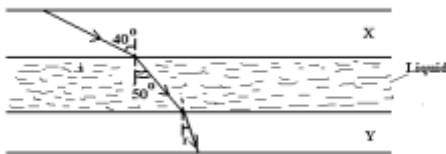
Using equation (i)

$$\tan \alpha = \frac{n_g}{n_l} = \frac{1.39}{1.33}$$

$$\alpha = \tan^{-1} (1.045) = 46.3^\circ$$

$$\begin{aligned} \text{The required angle of incidence} &= \alpha + 8^\circ \\ &= 54.3^\circ \end{aligned}$$

3. The figure below shows a liquid layer confined between two transparent plates X and Y of refractive index 1.54 and 1.44 respectively.



A ray of monochromatic light making an angle of  $40^\circ$  with the normal to the interface between media X and the liquid is refracted through an angle of  $50^\circ$  by the liquid. Find the

(i) refractive index of the liquid.

(ii) angle of refraction,  $r$  in the medium Y.

(iii) minimum angle of incidence in the medium X for which the light will not emerge from medium Y.

**Solution**

(i) Applying Snell's law at the plate X – liquid interface gives

$$n_x \sin 40^\circ = n_l \sin 50^\circ$$

$$1.54 \sin 40^\circ = n_l \sin 50^\circ$$

$$n_l = \frac{1.54 \sin 40^\circ}{\sin 50^\circ}$$

$$\therefore n_l = 1.29$$

(ii) Applying Snell's law at the liquid – plate Y interface gives

$$n_l \sin 50^\circ = n_y \sin r$$

$$1.29 \sin 50^\circ = 1.44 \sin r$$

$$\Rightarrow \angle r = 43.3^\circ$$

(iii) For light not to emerge from plate Y, it grazes the liquid – plate Y interface.

$$\Rightarrow \angle r = 90^\circ$$

Applying Snell's law at the liquid – plate Y interface gives

$$n_l \sin i_l = n_y \sin 90^\circ$$

$$1.29 \sin i_l = 1.44 \sin 90^\circ$$

$$\sin i_l = \frac{1.44}{1.29} \text{ -----(i)}$$

More over, applying Snell's law at the plate X – liquid interface gives

$$n_x \sin i_x = n_l \sin i_l$$

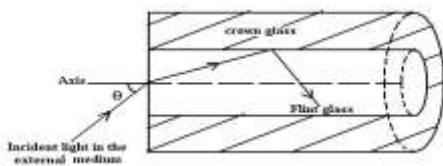
$$1.54 \sin i_x = 1.29 \sin i_l \text{ -----(ii)}$$

Substituting equation (i) in (ii) gives

$$1.54 \sin i_x = 1.29 \times \frac{1.44}{1.29}$$

$$\Rightarrow \angle i_x = 40.5^\circ$$

5.The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.



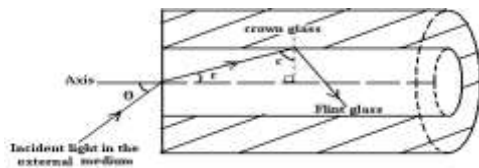
The refractive indices for flint glass, crown glass and the external medium are  $n_1$ ,  $n_2$  and  $n_3$  respectively. Show that a ray that enters the pipe is totally

reflected at the flint - crown glass interface provided  $\sin \theta = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$  where

$\theta$  is the maximum angle of incidence in the external medium.

Solution

Analysis for light to be totally reflected, it must be incident at a critical angle on the flint-crown glass interface



Applying Snell's law at the external medium-flint glass interface gives

$$n_3 \sin \theta = n_1 \sin r$$

but  $r + c = 90^\circ$

$$\therefore r = 90^\circ - c$$

$$\Rightarrow n_3 \sin \theta = n_1 \sin (90^\circ - c)$$

$$\therefore n_3 \sin \theta = n_1 \cos c$$

$$\Rightarrow \cos c = \frac{n_3 \sin \theta}{n_1} \text{ -----(i)}$$

Applying Snell's law at the flint-crown glass interface gives

$$n_1 \sin c = n_2 \sin 90^\circ$$

$$\Rightarrow \sin c = \frac{n_2}{n_1} \text{-----(ii)}$$

Using the trigonometrical relation  $\sin^2 c + \cos^2 c = 1$ , then

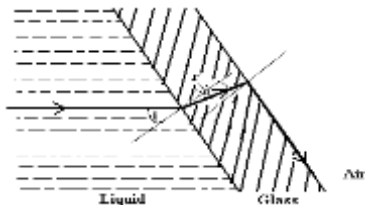
$$\left(\frac{n_2}{n_1}\right)^2 + \left(\frac{n_3 \sin \theta}{n_1}\right)^2 = 1$$

$$\text{Thus, } \sin \theta = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$$

## EXERCISE 2

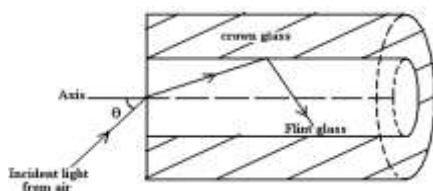
1. Explain the term total internal reflection and give three instances where it is applied.
2. With the aid of suitable ray diagrams, explain the terms critical angle and total internal reflection.
3. Show that the relation between the refractive index  $n$  of a medium and critical angle  $c$  for a ray of light traveling from the medium to air is given by  $n = \frac{1}{\sin c}$
4. Show that the critical angle,  $c$  at a boundary between two media when light travels from medium 1 to medium 2 is given by  $\sin c = \frac{n_2}{n_1}$  where  $n_1$  and  $n_2$  are the refractive indices of the media respectively.
5. Explain how a mirage is formed.
6. Explain briefly how sky radio waves travel from a transmitting station to a receiver.
7. Describe how you would determine the refractive index of the liquid using an air cell.

8)



In the figure above, a parallel sided glass slide is in contact with a liquid on one side and air on the other. A ray of light incident on glass slide from the liquid emerges in air along the glass-air interface. Derive an expression for the absolute refractive index,  $n$ , of the liquid in terms of the angle of incidence  $i$  in the liquid-medium.

(9) The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.

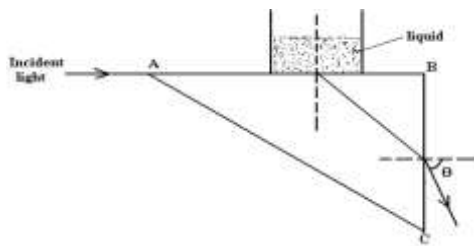


The refractive indices for flint glass and crown glass are  $n_1$  and  $n_2$  respectively. Show that a ray which enters the pipe is totally reflected at the flint-crown glass

interface provided  $\sin \theta = \sqrt{n_1^2 - n_2^2}$  where  $\theta$  is the maximum angle of incidence at the air-flint glass interface

10. A liquid of refractive index  $\mu_l$  is tapped in contact with the base of a right-angled prism of refractive index  $\mu_g$  by means of a transparent cylindrical pipe as shown.



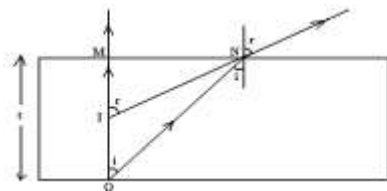


Show that a ray of light which is at a grazing incidence on the liquid-glass interface emerges in to air through face **BC** at an angle  $\theta$  below the horizontal provided

$$\mu_L = \sqrt{\mu_g^2 - \sin^2 \theta} . \text{ Hence find } \mu_L, \text{ if } \mu_g = 1.52 \text{ and } \theta = 47.4^\circ$$

## REAL AND APPARENT DEPTH

Consider an object **O** viewed normally from above through a parallel-sided glass block of refractive index  $n_g$  and thickness **t** as shown.



A ray from an object **O** normal to the glass surface at **M** passes un deviated. While the ray **ON** inclined at a small angle **i** to the normal is refracted at **N**. The observer above the glass block sees the image of the object **O** at **I**.

Applying Snell's law at **C** gives

$$n_g \sin i = n_a \sin r \text{-----(i).}$$

$$\text{From the diagram, } \sin i = \frac{MN}{ON} \text{ and } \sin r = \frac{MN}{IN}$$

$$\text{Equation (i) now becomes } n_g \frac{MN}{ON} = \frac{MN}{IN}$$

$$\Rightarrow n_g = \frac{ON}{IN}$$

Since angle **i** is very small, then  $ON \approx OM$  and  $IN \approx IM$

$$\Rightarrow n_g = \frac{OM}{IM}$$

From the diagram, **OM = real depth** and **IM = apparent depth**.

$$\text{Hence } n_g = \frac{\text{real depth}}{\text{apparent depth}} .$$

If the apparent displacement of the object **OI = d**, then **IM = (t - d)**.

$$\text{So that } n_g = \frac{OM}{IM} = \frac{t}{t - d} .$$

$$\text{Thus apparent displacement } d = \left( 1 - \frac{1}{n_g} \right) t$$

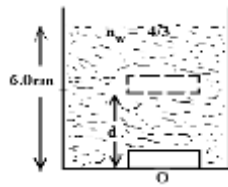
### NOTE

(i) The apparent displacement **d** of an object **O** is independent of the position of **O** below the glass block. Thus the same expression above gives the displacement of an object which is some distance in air below a parallel-sided glass block.

(ii) If there are different layers of different transparent materials resting on top of each other, the apparent position of the object at the bottom can be found by adding the separate displacements due to each layer.

### EXAMPLES:

1. An object at a depth of **6.0cm** below the surface of water of refractive index  $\frac{4}{3}$  is observed directly from above the water surface. Calculate the apparent displacement of the object



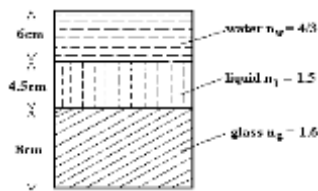
Using the relation  $d = \left(1 - \frac{1}{n_g}\right) t$  gives

$$d = \left(1 - \frac{3}{4}\right) \times 6$$

$$d = 1.5 \text{ cm}$$

2. A tank contains a slab of glass **8cm** and refractive index **1.6**. Above this is a depth of **4.5cm** of a liquid of refractive index **1.5** and upon this floats **6cm** of water of refractive index  $\frac{4}{3}$  calculate the apparent displacement of an object at the bottom of the tank to an observer looking down wards directly from above.

**Solution**



Apparent displacement  $d = d_w + d_l + d_g$

Using the relation  $d = \left(1 - \frac{1}{n}\right) t$

$$\therefore d = \left(1 - \frac{1}{n_w}\right) t_w + \left(1 - \frac{1}{n_l}\right) t_l + \left(1 - \frac{1}{n_g}\right) t_g$$

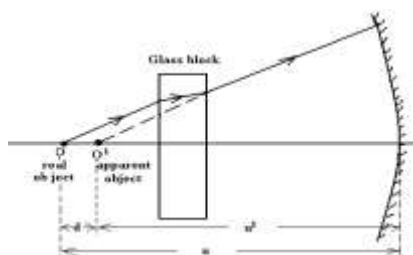
$$\Rightarrow d = \left(1 - \frac{3}{4}\right) \times 6 + \left(1 - \frac{1}{1.5}\right) \times 4.5 + \left(1 - \frac{1}{1.6}\right) \times 8$$

$$d = 1.5 + 1.5 + 3$$

$$d = 6\text{cm.}$$

3. A small object is placed **20cm** in front of a concave mirror of focal length **15cm**.

A parallel-sided glass block of thickness **6cm** and refractive index **1.5** is then placed between the mirror and the object. Find the shift in the position and size of the image



Consider the action of a concave mirror in the absence of a glass block

$$u = 20\text{cm} \text{ and } f = 15\text{cm}$$

using the mirror formula  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$  gives

$$v = \frac{fu}{u-f} = \frac{15 \times 20}{20-15} = 60\text{cm}$$

Thus in the absence of a glass block, image distance = 60cm

In this case, magnification  $m = \frac{v}{u} = \frac{60}{20} = 3$

Consider the action of a glass block

$$\text{Apparent displacement } d = \left(1 - \frac{1}{n_g}\right)t = \left(1 - \frac{1}{1.5}\right) \times 6 = 2\text{cm}$$

Thus in the presence of a glass block, object distance  $u^1 = (20 - 2)\text{ cm} = 18\text{cm}$

"The object is displaced and it appears to be 18cm in front of the mirror"

Consider the action of a concave mirror in the presence of a glass block

$$u^1 = 18\text{cm and } f = 15\text{cm}$$

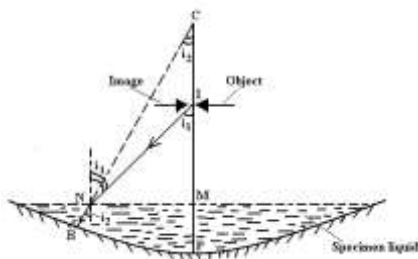
using the mirror formula  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$  gives

$$v^1 = \frac{fu^1}{u^1 - f} = \frac{15 \times 18}{18-15} = 90\text{cm}$$

$\therefore$  The required shift in the image position =  $v^1 - v = (90 - 60)\text{ cm} = 30\text{cm}$

The magnification now becomes  $m^1 = \frac{v^1}{u^1} = \frac{90}{18} = 5$

#### DETERMINATION OF REFRACTIVE INDEX OF A LIQUID USING A CONCAVE MIRROR METHOD.



A small quantity of the liquid under test is poured into a concave mirror of known radius of curvature  $r$ . An object pin is moved along the principal axis of the mirror until it coincides with its image at I. The distance  $IP$  is noted. The required refractive index  $n_l = \frac{r}{IP}$ .

IP.

#### PROOF

For refraction at N,  $n_l \sin i_2 = n_a \sin i_1$  ----- (i)

From the diagram,  $\sin i_2 = \frac{NM}{NC}$  and  $\sin i_1 = \frac{MN}{NI}$

Equation (i) now becomes

$$n_l \frac{NM}{NC} = \frac{NM}{NI}$$

On simplifying,  $n_l = \frac{NC}{NI}$

But N is very close to M hence  $NC \approx MC$  and  $NI \approx MN$

$$\Rightarrow n_l = \frac{MC}{MI}$$

Also for a small quantity of the liquid, **M** is close to **P**  $\Rightarrow MC \approx CP = r$ , and  $MI \approx IP$ .

$$\text{Thus } n_l = \frac{r}{IP}.$$

NOTE :

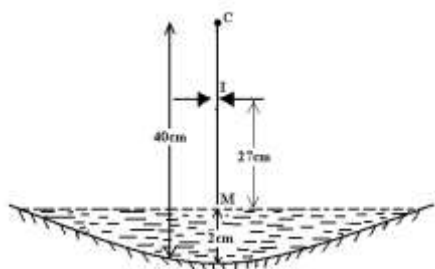
(i) If the specimen liquid is of reasonable quantity, then its depth **d** can not be

ignored. In this case,  $n_l = \frac{MC}{MI} = \frac{r-d}{MI}$

(ii) if the radius of curvature **r** of the concave mirror is not known, first determine it using the method discussed in the previous section.

#### EXAMPLES:

1. A liquid is poured in to a concave mirror to a depth of **2.0cm**. An object held above the liquid coincides with its own image when its **27.0cm** above the liquid surface. If the radius of curvature of the mirror is **40.0cm**, calculate the refractive index of the liquid.

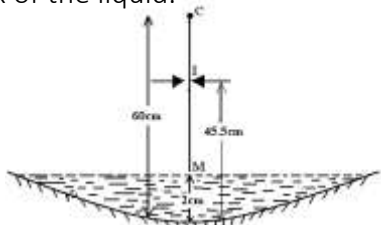


Using the relation  $n_l = \frac{r-d}{MI}$

$$= \frac{40 - 2}{27}$$

$$\therefore n_l = 1.4$$

2. A liquid is poured into a concave mirror to a depth of **2.0cm**. An object held above the liquid coincides with its image when it is **45.5cm** from the pole of the mirror. If the radius of curvature of the mirror is **60.0cm**, calculate the refractive index of the liquid.



Using the relation  $n_l = \frac{r-d}{MI}$

$$= \frac{60 - 2}{45.5 - 2}$$

$$\therefore n_l = 1.33$$

3. A small concave mirror of focal length **8cm** lies on a bench and a pin is moved vertically above it. At what point will the pin coincide with its image if the mirror is filled with water of refractive index **4/3**.

**Solution**

**ANALYSIS**

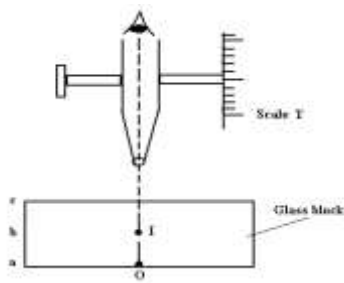
For a small concave mirror, the quantity of water is small that its depth **d** can be ignored

Using the relation  $n_l = \frac{r}{IP}$  Where  $r = 2f = 2 \times 8 = 16\text{cm}$

$$\Rightarrow IP = \frac{r}{n_l} = \frac{16}{4/3} = 12\text{cm}$$

Therefore the pin coincided with its image at a height of **12cm** above the mirror

## MEASUREMENT OF REFRACTIVE INDEX OF A GLASS BLOCK BY APPARENT DEPTH METHOD.



A vertically traveling microscope having a graduated scale **T** besides it is focused on lycopodium particles placed at **O** on a sheet of white paper. The scale reading **a** on **T** is noted. A glass block whose refractive index is required is placed on a paper and the microscope is raised until the particles are refocused at **I**. The scale reading **b** is again noted. Lycopodium particles are then poured on top of the glass block and the microscope is re-raised until the particles are again refocused. The new scale reading **c** is then noted. The refractive index of the block can then be calculated from

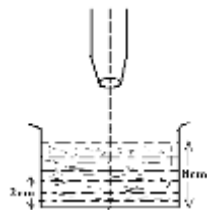
$$n = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{|c - a|}{|c - b|}$$

### NOTE

The refractive index of a liquid can be found by focusing on a sand particle at the bottom of an empty container and the scale reading is noted. The specimen liquid is then put in the container and the traveling microscope is refocused on the sand giving a scale reading **b**. Finally the traveling microscope is focused on the liquid surface giving a scale reading **c**. Thus  $n_L = \frac{|c - a|}{|c - b|}$

### EXAMPLE:

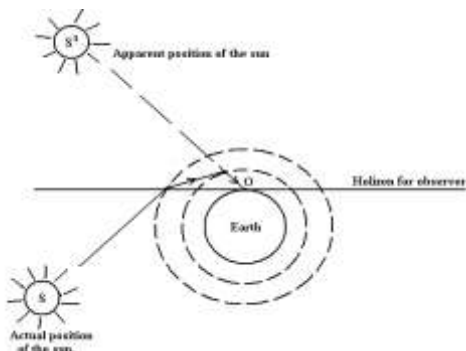
A microscope is focused on a mark at the bottom of the beaker. Water is poured in to the beaker to a depth of **8cm** and it is found necessary to raise the microscope through a vertical distance of **2cm** to bring the mark again in to focus. Find the refractive index of water.



Using the relation  $n_g = \frac{\text{real depth}}{\text{apparent depth}} = \frac{8}{8 - 2}$

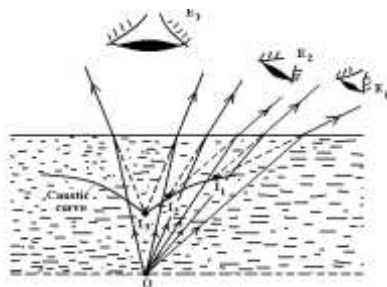
$\Rightarrow n_g = 1.33$  (to 2 decimal places)

## OBSERVATION OF SUNLIGHT SITUATED BELOW THE HORIZON.



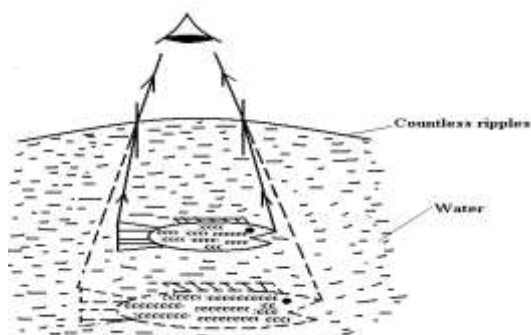
Light from the sun **S** is successively refracted from the different layers of the atmosphere due to the different optical densities they have. To an observer **O** on the earth's surface, sunlight appears to have come from point **S'**.

## THE APPARENT SHAPE OF THE BOTTOM OF A POOL OF WATER



When the observer moves the eye from **E<sub>1</sub>** towards **E<sub>3</sub>** the apparent position of **O** moves from **I<sub>1</sub>** to **I<sub>3</sub>** along a curve due to refracted rays bending away from the normal as shown above. Thus the bottom of the pool of water appears curved, being shallower near the edge than at the centre.

## THE APARENT SIZE OF A FISH SITUATED IN WATER.



A large surface of water is not completely flat but consists of countless ripples whose convex surface on air acts as a convex lens of long focal length. In consequence the fish is within the focal length of the lens hence it appears magnified to an observer viewing it from above.

## EXERCISE

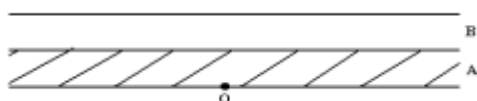
1. Show that for an object viewed normally from above through a parallel sided glass block, the refractive index of the glass material is given by

$$n_g = \frac{\text{real depth}}{\text{Apparent depth}}$$

2. Derive an expression for the apparent displacement of an object when viewed normally through a parallel sided glass block.

3. A vessel of depth **2d cm** is half filled with a liquid of refractive index  **$\mu_1$** , and the upper half is occupied by a liquid of refractive index  **$\mu_2$** . Show that the apparent depth of the vessel, viewed perpendicularly is  $\left( \frac{1}{\mu_1} + \frac{1}{\mu_2} \right) d$

4. Two parallel sided blocks **A** and **B** of thickness **4.0cm** and **5.0cm** respectively are arranged such that **A** lies on an object **O** as shown in the figure below



Calculate the apparent displacement of **O** when observed directly from above, if the refractive indices of **A** and **B** are **1.52** and **1.66**.

**5.** A tank contains liquid **A** of refractive index **1.4** to a depth of **7.0cm**. Upon this floats **9.0cm** of liquid **B**. If an object at the bottom of the tank appears to be **11.0cm** below the top of liquid **B** when viewed directly above from, calculate the refractive index of liquid **B**.

**6.** Describe how the refractive index of a small quantity of a liquid can be determined using a concave mirror.

**7.** Describe how the refractive index of a glass block can be determined using the apparent depth method.

**8.** A small liquid quantity is poured into a concave mirror such that an object held above the liquid coincides with its image when it is at a height **h** from the pole of the mirror. If the radius of curvature of the mirror is **r**, show with the aid of a suitable illustration, that the refractive index of the liquid.  $n = \frac{r}{h}$

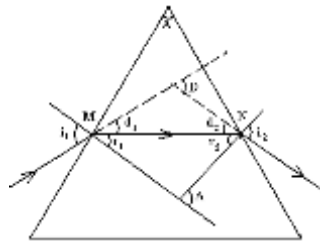
**9.** Explain how light from the sun reaches the observer in the morning before the sun appears above the horizon

**10.** Explain the apparent shape of the bottom of a pool of water to an observer at the bank of the pool.

**11.** Explain why a fish appears bigger in water than its actual size when out of water.

## DEVIATION OF LIGHT THROUGH A PRISM

The angle of deviation caused by the prism is the angle between the incident ray and the emergent ray. Consider a ray of light incident in air on a prism of refracting angle **A** and finally emerges into air as shown.



From the diagram above, MS and NS are normals at the points of incidence and emergence of the ray respectively.

$$\therefore \angle MPN + \angle MSN = 180^\circ$$

$$\text{Thus } \angle NST = \angle MPN = A$$

Suppose  **$i_1, r_1$**  and  **$i_2, r_2$**  represents angles of incidence and refraction at faces M and N respectively, then

$$\text{From geometry of } \triangle MNS, \quad r_1 + r_2 = A \text{ ----- (i)}$$

$$\text{Total deviation } D = d_1 + d_2 \quad \text{where } d_1 = i_1 - r_1 \quad \text{and} \quad d_2 = i_2 - r_2$$

$$\Rightarrow D = i_1 - r_1 + i_2 - r_2$$

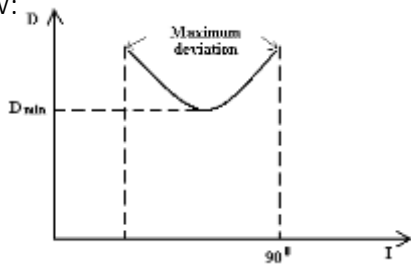
$$\text{On simplifying, } D = i_1 + i_2 - (r_1 + r_2) \text{ ----- (ii)}$$

Combining equation (i) and (ii) gives

$$D = i_1 + i_2 - A$$

### NOTE:

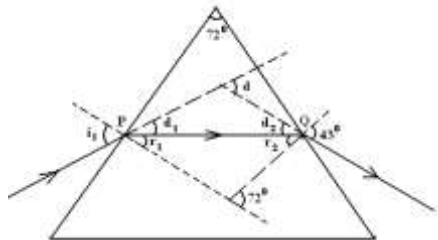
Experiments show that as the angle of incidence **i** is increased from zero, the deviation **D** reduces continuously up to a minimum value of deviation  **$D_{min}$**  and then increases to a maximum value as the angle of incidence is increased as shown below:



### EXAMPLE:

1. A ray of light is incident on a prism of refracting angle  **$72^\circ$**  and refractive index of **1.3**. The ray emerges from the prism at  **$43^\circ$** . Find

- (i) the angle of incidence.
- (ii) the deviation of the ray.



(i) At **P**, Snell's law becomes .

$$n_a \sin 43^\circ = 1.3 \sin r_2$$

$$\therefore r_2 = 31.64^\circ$$

$$\text{But } r_1 + r_2 = 72^\circ$$

$$\begin{aligned} \Rightarrow r_1 &= 72^\circ - r_2 \\ &= 72^\circ - 31.64^\circ \end{aligned}$$



$$\therefore r_1 = 40.36^\circ$$

At Q, Snell's law becomes

$$n_a \sin i_1 = 1.3 \sin 40.36^\circ$$

$$\therefore i_1 = 57.34^\circ$$

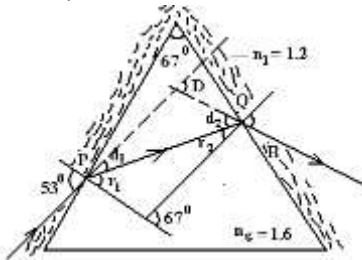
(ii) Total Deviation  $D = d_2 + d_1$

where  $d_1 = i_1 - r_1$  and  $d_2 = i_2 - r_2$

$$\Rightarrow D = (43^\circ - 31.64^\circ) + (57.34^\circ - 40.36^\circ)$$

$$\therefore D = 28.34^\circ$$

2. A prism of refracting angle  $67^\circ$  and refractive index of 1.6 is immersed in a liquid of refractive index 1.2. If a ray of light traveling through the liquid makes an angle of incidence of  $53^\circ$  at the left face of the prism, Determine the total deviation of the ray.



Total Deviation  $D = d_2 + d_1$

where  $d_1 = i_1 - r_1$  and  $d_2 = i_2 - r_2$

$$\Rightarrow D = (53^\circ - r_1) + (i_2 - r_2) \text{ -----(i)}$$

At P, Snell's becomes

$$1.2 \sin 53^\circ = 1.6 \sin r_1$$

$$\therefore r_1 = 36.8^\circ$$

$$\text{But } r_1 + r_2 = 67^\circ$$

$$\Rightarrow r_2 = 67^\circ - r_1$$

$$= 67^\circ - 36.8^\circ$$

$$\therefore r_2 = 30.2^\circ$$

At Q, Snell's law becomes

$$1.6 \sin 30.2^\circ = 1.2 \sin i_2$$

$$i_2 = 42.12^\circ$$

Substituting for  $r_1$ ,  $i_2$ , and  $r_2$  in equation (i) gives

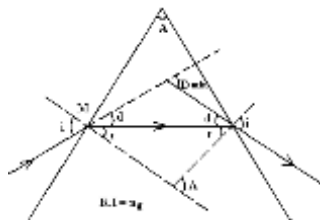
$$D = (53^\circ - 36.8^\circ) + (42.12^\circ - 30.2^\circ)$$

$$D = 28.12^\circ$$

## MINIMUM DEVIATION OF LIGHT BY A PRISM

.At minimum deviation, light passes **symmetrically** through the prism. That is to say, the angle of incidence is equal to the angle of emergence.

Consider a ray on one face of the prism at an angle  $i_1$  and leaves it at an angle  $i_2$  to the normal as shown



For minimum deviation,  $i_1 = i_2 = i$  and  $r_1 = r_2 = r$ .

From the diagram,  $D_{\min} = d + d$

$$D_{\min} = 2d \quad \text{where } d = i - r$$

$$\Rightarrow D_{\min} = 2i - 2r \text{-----(a)}$$

Moreover,  $r + r = A$ .

$$\Rightarrow 2r = A \quad \text{OR} \quad r = \frac{A}{2} \text{-----(b)}$$

Combining equation (a) and (b) gives.

$$D_{\min} = 2i - A$$

$$\Rightarrow i = \frac{D_{\min} + A}{2} \text{-----(c)}$$

At M snell's law becomes

$$n_a \sin i = n_g \sin r$$

$$\Rightarrow n_g = \frac{n_a \sin i}{\sin r} \text{-----(d)}$$

substituting equation (b) and (c) in (d) gives

$$n_g = n_a \frac{\sin\left(\frac{D_{\min} + A}{2}\right)}{\sin \frac{A}{2}} \text{-----(e)}$$

Since  $n_a = 1$ ,

$$\Rightarrow n_g = \frac{\sin\left(\frac{D_{\min} + A}{2}\right)}{\sin \frac{A}{2}}$$

NOTE ;

Equation (e) suggests that if the prism was surrounded by a medium of refractive index  $n_1$ , then at minimum deviation

$$n_g = n_1 \frac{\sin\left(\frac{D_{\min} + A}{2}\right)}{\sin \frac{A}{2}}$$

EXAMPLES:

1. Calculate the angle of incidence at minimum deviation for light passing through a Prism of refracting angle **70°** and refractive index of **1.65**.

Solution

$$\text{Using } n_g = n_a \frac{\sin\left(\frac{D_{\min} + A}{2}\right)}{\sin \frac{A}{2}} \quad \text{where } n_g = 1.65, A = 70^\circ \text{ and } n_a = 1$$

$$\Rightarrow 1.65 = \frac{\sin\left(\frac{D_{\min} + 70^\circ}{2}\right)}{\sin 70^\circ/2}$$

On solving,  **$D_{\min} = 72.33^\circ$**

$$\text{The required angle of incidence } i = \frac{D_{\min} + A}{2} = \frac{72.33^\circ + 70^\circ}{2} = \mathbf{71.165^\circ}$$

2. An equilateral glass prism of refractive index **1.5** is completely immersed in a liquid of refractive index **1.3**. If a ray of light passes symmetrically through the prism, calculate the:

(i) angle of deviation of the ray.

(ii) angle of incidence

**ANALYSIS:**

(a) For an equilateral prism, its refracting angle  $A = 60^\circ$

(b) If the ray passes through the prism symmetrically, then the angle of deviation is minimum

(i) Using  $n_g = n_1 \frac{\sin\left(\frac{D_{\min} + A}{2}\right)}{\sin\frac{A}{2}}$  where  $n_g = 1.5$ ,  $A = 60^\circ$  and  $n_1 = 1.3$

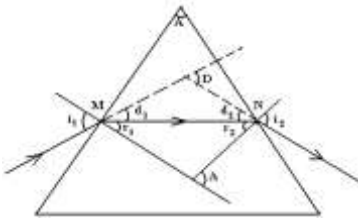
$$\Rightarrow 1.5 = 1.3 \frac{\sin\left(\frac{D_{\min} + 60^\circ}{2}\right)}{\sin 60^\circ/2}$$

On solving,  $D_{\min} = 10.47^\circ$

(ii) The required angle of incidence  $i = \frac{D_{\min} + A}{2} = \frac{10.47^\circ + 60^\circ}{2} = 35.235^\circ$

#### DEVIATION OF LIGHT BY A SMALL ANGLE PRISM

The small refracting angles of this prism causes the angle  $i_1$ ,  $r_1$ ,  $r_2$  and  $i_2$  to be small such that  $\sin i_1 \approx i_1$ ,  $\sin r_1 \approx r_1$ ,  $\sin r_2 \approx r_2$  and  $\sin i_2 \approx i_2$ .



From the diagram,  $D = d_1 + d_2$

but  $d_1 = i_1 - r_1$  and  $d_2 = i_2 - r_2$

$$\Rightarrow D = (i_1 - r_1) + (i_2 - r_2)$$

On simplifying  $D = i_1 + i_2 - (r_1 + r_2)$

but  $r_1 + r_2 = A$

$$\therefore D = i_1 + i_2 - A \text{-----(a)}$$

At M Snell's law becomes.

$$n_a \sin i_1 = n \sin r_1$$

For small angles this gives  $i_1 = nr_1$ ----- (b)

Similarly at N Snell's law becomes  $i_2 = nr_2$ ----- (c)

Substituting equation (b) and (c) in (a) gives

$$D = nr_1 + nr_2 - A$$

$$D = n(r_1 + r_2) - A$$

but  $r_1 + r_2 = A$

$$\Rightarrow D = nA - A$$

$$\therefore D = (n - 1)A$$

$\therefore$  The deviation produced by a small angle prism is independent of the magnitude of the small angle of incidence on the prism. (ie: All rays entering a small-angle prism at small angles of incidence suffer the same deviation)

NOTE:

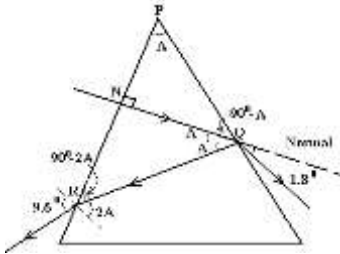
The result  $D = (n - 1)A$  will later be used in developing lens theory.

#### EXAMPLES:

A ray of light that falls normally upon the first face of a glass prism of a small refracting angle under goes a partial refraction and reflection at the second face of the prism. The refracted ray is deviated through an angle  $1.8^\circ$  and the reflected ray makes an angle of  $9.6^\circ$  with the incident ray after emerging from the prism through its first face. Calculate the refracting angle of the prism and its refractive index of the glass material.

Solution

Let  $A$  be the required refracting angle of the prism as shown



Consider the deviation suffered by the incident light

$$D = (n - 1) A \quad \text{where } D = 1.8^\circ$$

$$\Rightarrow 1.8^\circ = (n - 1) A \text{ -----(i)}$$

From  $\triangle PQN$ ,  $\angle PQN = 90^\circ - A$

$\Rightarrow$  At Q, the angle of incidence =  $A$

From  $\triangle NQR$ ,  $\angle QRN = 90^\circ - 2A$

$\Rightarrow$  At R, the angle of incidence =  $2A$

$\therefore$  At R, Snell's becomes  $n_a \sin 9.6^\circ = n \sin 2A$

For small angles,  $\sin 9.6^\circ \approx 9.6^\circ$  and  $\sin A \approx 2A$

$$\Rightarrow 9.6^\circ = 2nA \text{ -----(ii)}$$

Equation (i)  $\div$  Equation (ii) gives

$$\frac{1.8^\circ}{9.6^\circ} = \frac{(n - 1) A}{2nA}$$

$$\Rightarrow 3.6^\circ n = 9.6^\circ (n - 1)$$

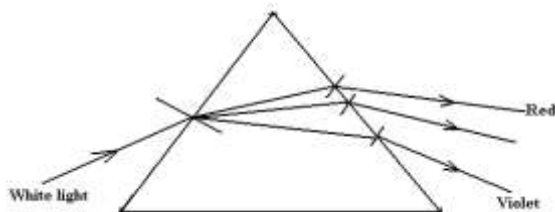
Thus  $n = 1.6$

Equation (i) now becomes  $1.8^\circ = (1.6 - 1) A$

$$\therefore A = 3^\circ$$

## DISPERSION OF WHITE LIGHT BY A TRANSPARENT MEDIUM

Dispersion of white light is the separation of white light into its component colours by a transparent medium due to their speed differences in the medium.



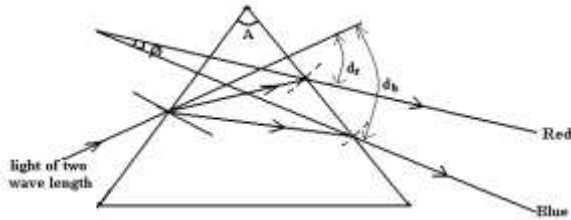
When white light falls on a transparent medium, its different component colours travel with different speeds through the medium. They are therefore deviated by different amounts on refraction at the surface of the medium and hence dispersion.

#### NOTE :

(i) White light is a mixture of various colours. This is called the spectrum of white light.

(ii) The spectrum of white light consists of red, orange, yellow, green, blue, indigo and violet light bands. On refraction, violet is the most refracted colour away from the normal (violet is the most deviated colour ) while red is least deviated

(iii) When light of two wavelengths say red and blue light is incident at a small angle on a small angle prism of refracting angle  $A$  having refractive indices of  $n_r$  and  $n_b$  for the two wave lengths respectively, then the two wave lengths are deviated as shown below.



The deviation of red and blue light is given by  $d_r = (n_r - 1)A$   
 $d_b = (n_b - 1)A$ .

The quantity  $\phi = d_b - d_r$  is called the **Angular separation (Angular dispersion)** produced by the prism.

$$\Rightarrow \phi = (n_b - 1)A - (n_r - 1)A$$

$$\text{on simplifying } \phi = (n_b - n_r)A$$

#### EXAMPLES:

1. Light of two wave length is incident at a small angle on a thin prism of refracting angle  $5^\circ$  and refractive index of 1.52 and 1.48 for the two wave lengths. Find the angular separation of the two wave lengths after refraction by the prism.

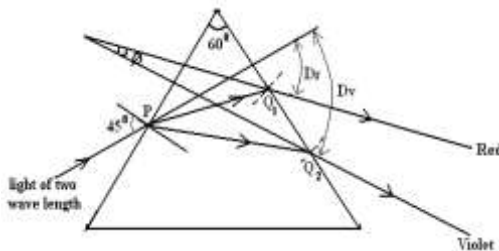
#### Solution

For a small prism, Angular separation  $\phi = (n_1 - n_2)A$

$$\phi = (1.52 - 1.48) \times 5^\circ$$

$$\Rightarrow \phi = 0.2^\circ$$

2. A glass prism with refracting angle  $60^\circ$  has a refractive index of 1.64 for red light and 1.66 for violet light. Calculate the angular separation of the red and violet rays which emerge from the prism when a ray of white light is incident on the prism at an angle of  $45^\circ$



#### Case I:

Consider the deviation suffered by red light

At P, Snell's law becomes .

$$n_a \sin 45^\circ = 1.64 \sin r_1$$

$$\therefore r_1 = 25.54^\circ$$

$$\text{But } r_1 + r_2 = 60^\circ$$

$$\Rightarrow r_2 = 60^\circ - r_1$$

$$= 60^\circ - 25.54^\circ$$

$$\therefore r_2 = 34.46^\circ$$

At Q1, Snell's law becomes

$$n_a \sin i_2 = 1.64 \sin 34.46^\circ$$

$$\therefore i_2 = 68.13^\circ$$

Total Deviation  $D_r = d_2 + d_1$  where  $d_1 = i_1 - r_1$  and  $d_2 = i_2 - r_2$

$$\Rightarrow D_r = (45^\circ - 25.54^\circ) + (68.13^\circ - 34.46^\circ)$$

$$\therefore D_r = 53.13^\circ.$$

### Case II:

Consider the deviation suffered by violet light

At P, Snell's law becomes .

$$n_a \sin 45^\circ = 1.66 \sin r_1$$

$$\therefore r_1 = 25.21^\circ$$

$$\text{But } r_1 + r_2 = 60^\circ$$

$$\Rightarrow r_2 = 60^\circ - r_1$$

$$= 60^\circ - 25.21^\circ$$

$$\therefore r_2 = 34.79^\circ$$

At Q, Snell's law becomes

$$n_a \sin i_2 = 1.66 \sin 34.79^\circ$$

$$\therefore i_2 = 71.28^\circ.$$

Total Deviation  $D_v = d_2 + d_1$  where  $d_1 = i_1 - r_1$  and  $d_2 = i_2 - r_2$

$$\Rightarrow D_v = (45^\circ - 25.21^\circ) + (71.28^\circ - 34.79^\circ)$$

$$\therefore D_v = 56.28^\circ.$$

Thus required angular separation  $\phi = D_v - D_r$

$$\phi = 56.28^\circ - 53.13^\circ$$

$$\Rightarrow \phi = 3.15^\circ$$

## APPEARANCE OF WHITE LIGHT PLACED IN WATER

### OBSERVATION:

A coloured spectrum is seen inside the water surface with violet on top and red down.

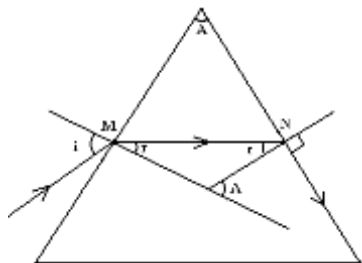
### EXPLANATION:

The different component colours of white light travel with different speeds through water. They are therefore deviated by different amounts on refraction at the water surface. Hence different coloured images are formed at different points inside the water surface with a violet coloured image on top.

## GRAZING PROPERTY OF LIGHT RAYS AS APPLIED TO PRISMS.

If a ray of light is either such that the incident angle or the emergent angle is equal to  $90^\circ$  to the normal of the prism, then the ray is said to graze the refracting surface of the prism.

Consider a ray of light incident at an angle  $i$  on a glass prism of refracting angle  $A$  situated in air with the emergent light grazing the other refracting surface of the prism as shown.



From the diagram,  $r + c = A$

$$\therefore r = A - c \text{-----(a)}$$

At M Snell's law becomes

$$n_a \sin i = n_g \sin r \text{-----(b)}$$

Substituting equation (a) in (b) gives

$$\sin i = n_g \sin (A - c)$$

$$\Rightarrow \sin i = n_g (\sin A \cos C - \sin C \cos A) \text{-----(c)}$$

At N, Snell's law becomes

$$n_g \sin c = n_a \sin 90^\circ.$$

$$\therefore \sin c = \frac{1}{n_g}$$

$$\Rightarrow \cos c = \sqrt{1 - \sin^2 c} = \sqrt{1 - \frac{1}{n_g^2}} = \frac{\sqrt{n_g^2 - 1}}{n_g}$$

Substituting  $\sin c$  and  $\cos c$  in equation c gives

$$\sin i = n_g \left( \sin A \times \frac{\sqrt{n_g^2 - 1}}{n_g} - \frac{1}{n_g} \cos A \right)$$

On simplifying we have

$$\sqrt{n_g^2 - 1} = \frac{\sin i + \cos A}{\sin A}$$

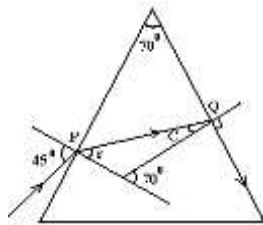
Squaring both sides and simplifying for  $n_g$  gives

$$n_g = \sqrt{1 + \left( \frac{\sin i + \cos A}{\sin A} \right)^2}$$

Knowing the angles  $i$  and  $A$ , the refractive index  $n_g$  of a material of a prism can be determined.

### EXAMPLES

1. Monochromatic light is incident at an angle of  $45^\circ$  on a glass prism of refracting angle  $70^\circ$  in air. The emergent light grazes the other refracting surface of the prism. Find the refractive index of the glass material.



At P, Snell's law becomes

$$n_a \sin 45^\circ = n_g \sin r \quad \text{-----(a)}$$

From the diagram,  $r + c = 70^\circ$

$$\Rightarrow r = 70^\circ - c \quad \text{-----(b)}$$

Substituting equation (b) in (a) gives

$$\sin 45^\circ = n_g \sin (70^\circ - c) \quad \text{-----(c)}$$

At Q, Snell's law becomes

$$n_g \sin c = n_a \sin 90^\circ$$

$$\Rightarrow n_g = \frac{1}{\sin c} \quad \text{-----(d)}$$

Substituting equation (d) in (c) gives

$$\sin 45^\circ = \frac{\sin (70^\circ - c)}{\sin c}$$

$$\sin 45^\circ \sin c = \sin 70^\circ \cos c - \sin c \cos 70^\circ$$

$$\Rightarrow (\sin 45^\circ + \cos 70^\circ) \sin c = \sin 70^\circ \cos c$$

Dividing  $\cos c$  through out gives

$$\tan c = \frac{\sin 70^\circ}{\sin 45^\circ + \cos 70^\circ}$$

$$\Rightarrow c = 41.9^\circ$$

$$\text{Equation (d) now becomes } n_g = \frac{1}{\sin 41.9^\circ}$$

$$\therefore n_g = 1.497$$

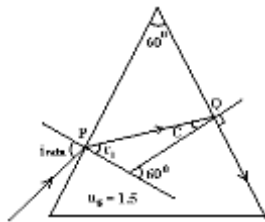
NOTE

$$\text{For grazing condition, you may as well use the relation } n_g = \sqrt{1 + \left( \frac{\sin i + \cos A}{\sin A} \right)^2}$$

$$\Rightarrow n_g = \sqrt{1 + \left( \frac{\sin 45^\circ + \cos 70^\circ}{\sin 70^\circ} \right)^2}$$

$$\therefore n_g = 1.498$$

2..A ray of light is incident on one refracting face of a prism of refractive index 1.5 and refracting angle  $60^\circ$ . Calculate the minimum angle of incidence for the ray to emerge through the second refracting face.



ANALYSIS

for minimum angle of incidence, the emergent ray grazes the second refracting face.

At Q, Snell's law becomes

$$1.5 \sin c = n_a \sin 90^\circ$$

$$\Rightarrow c = 41.8^\circ$$

$$\text{But } r + c = 60^\circ$$

$$\Rightarrow r = 60^\circ - c$$

$$= 60^\circ - 41.8^\circ$$

$$\therefore r = 18.2^\circ$$

At P, Snell's law becomes

$$1.5 \sin 18.2^\circ = n_a \sin i_{\min}$$

$$\therefore i_{\min} = 27.9^\circ$$

NOTE

$$\text{For grazing condition, you may as well use the relation } n_g = \sqrt{1 + \left( \frac{\sin i + \cos A}{\sin A} \right)^2}$$

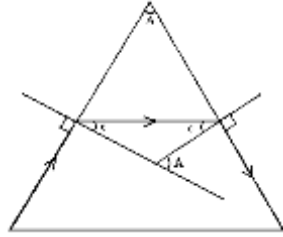
$$\Rightarrow 1.5 = \sqrt{1 + \left( \frac{\sin i_{\min} + \cos 60^\circ}{\sin 60^\circ} \right)^2}$$

$$\text{On simplifying, } i_{\min} = 27.9^\circ$$

## LIMITING ANGLE OF THE PRISM



This is the maximum refracting angle of the prism for which the emergent ray grazes the second refracting surface. Suppose the incident ray grazes the first refracting surface then the limiting angle  $A$  is given by  $A = 2c$  where  $c$  is the critical angle of the glass air interface as shown.



#### EXERCISE:

1. (i) Obtain an expression relating the deviation of a ray of light by the prism to the refracting angle and the angles of incidence and emergence.

(ii) The deviation of a ray of light incident on the first face of a  $60^\circ$  glass prism at an angle of  $45^\circ$  is  $40^\circ$ . Calculate the angle of emergence of a ray on the second face of the prism [ Ans  $i_2 = 65^\circ$  ]

(iii) A prism of refractive index  $1.64$  is immersed in a liquid of refractive index  $1.4$ . A ray of light is incident on one face of the prism at an angle of  $40^\circ$ . If the ray emerges at an angle of  $29^\circ$ , determine the angle of the prism.

[Answer:  $57.7^\circ$ ]

2. (i) For a ray of light passing through the prism, what is the condition for minimum deviation to occur?

(ii) Derive an expression for the refractive index of a prism in terms of the refracting angle,  $A$ , and the angle of minimum deviation  $D$ .

(iii) A glass prism of refractive index  $n$  and refracting angle,  $A$ , is completely immersed in a liquid of refractive index  $n_i$ . If a ray of light that passes symmetrically through the prism is deviated through an angle  $\phi$ , Show that

$$\frac{n_i}{n} = \frac{\sin \frac{A}{2}}{\sin \left( \frac{\phi + A}{2} \right)}$$

3.(a) A glass prism with refracting angle  $60^\circ$  is made of glass whose refractive indices for red and violet light are respectively  $1.514$  and  $1.530$ . A ray of white light is set incident on the prism to give a minimum deviation for red light.

Determine the:

- (i) angle of incidence of the light on the prism.
- (ii) angle of emergence of the violet light.
- (iii) angular width of the spectrum.

(b) A certain prism is found to produce a minimum deviation of  $51^\circ$ . While it produces a deviation of  $62.8^\circ$  for a ray of light incident on its first face at an angle of  $40.1^\circ$  and emerges through its second face at an angle of  $82.7^\circ$ .

Determine the:

- (i) refracting angle of the prism.
- (ii) angle of incidence at minimum deviation.
- (iii) refractive index of the material of the prism.

[ Ans (i)  $60^\circ$  (ii)  $55.5^\circ$  (iii)  $1.648$  ]

4. (i) A ray of monochromatic light is incident at a small angle of incidence on a small angle prism in air. Obtain the expression  $D = (n - 1)A$  for the deviation of light by the prism.

(ii) A glass prism of small angle,  $A$ , and refractive index  $n_g$  and is completely immersed in a liquid of refractive index  $n_L$ . Show that a ray of light passing through the prism at a small angle of incidence suffers a deviation given by

$$D = \left( \frac{n_g}{n_L} - 1 \right) A$$

5. Explain why white light is dispersed by a transparent medium.

6. Light of two wave length is incident at a small angle on a thin prism of refracting angle  $5^\circ$  and refractive index of 1.52 and 1.48 for the two wave lengths. Find the angular separation of the two wave lengths after refraction by the prism.

[ Ans  $\phi = 0.2^\circ$  ]

7. A point source of white light is placed at the bottom of a water tank in a dark room. The light from the source is observed obliquely at the water surface. Explain what is observed.

8. Monochromatic light is incident at an angle  $\phi$  on a glass prism of refracting angle,  $A$ , situated in air. If the emergent light grazes the other refracting surface of the prism, Show that the refractive index,  $n_g$ , of the prism material is given by

$$n_g = \sqrt{1 + \left( \frac{\sin i + \cos A}{\sin A} \right)^2}$$

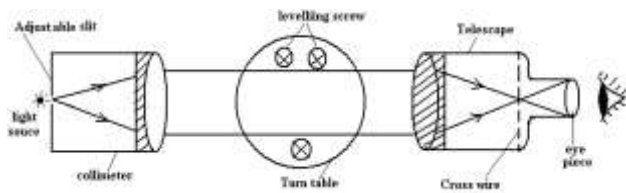
9. A ray of light is incident at angle of  $30^\circ$  on a prism of refractive index 1.5 .calculate the limiting angle of the prism such that the ray does not emerge when it meets the second face.

[ Ans  $A = 61.3^\circ$  ]

## A SPECTROMETER

It is an instrument used to measure accurate determination of deviation of a parallel beam of light which has passed through a prism. This provides a mean of studying optical spectra and measurement of refractive indices of glass in form of a prism.

It consists of a collimator, a telescope, and a turn table on which the prism is placed as shown.



Before the spectrometer is put in to use, 3 adjustments must be made onto it and these include,

- (i) The collimator is adjusted to produce parallel rays of light.
- (ii) The turntable is leveled.
- (iii) The telescope is adjusted to receive light from the collimator on its crosswire.

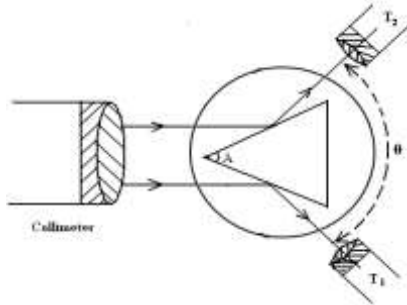
## MEASUREMENT OF THE REFRACTING ANGLE “A” OF THE PRISM

The collimator is adjusted to produce parallel rays of light.

The turn table is leveled.

The telescope is adjusted to receive light from the collimator on its cross wire.

The prism is placed on the turn table with its refracting angle facing the collimator as shown.



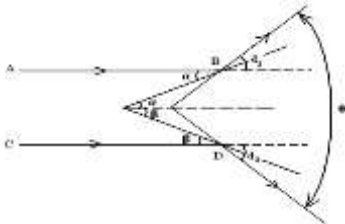
With the table fixed, the telescope is moved to position  $T_1$  to receive the light from the collimator on its cross wire.

This position  $T_1$  is noted and the telescope is turned to a new position  $T_2$  to receive light on its cross wire. The angle  $\theta$  between  $T_1$  and  $T_2$

is measured. The prism angle  $A$  is given by  $A = \frac{\theta}{2}$

### PROOF OF THE RELATION

Consider a parallel beam of light incident on to a prism of refracting angle  $A$  making glancing angles  $\alpha$  and  $\beta$  as shown.



From the geometry,  $\alpha + \beta = A$ -----(i).

Deviation  $d_1$  of ray  $AB = 2\alpha$

Deviation  $d_2$  of ray  $CD = 2\beta$ .

Total deviation  $\theta = d_1 + d_2$

$$= 2\alpha + 2\beta$$

$$= 2(\alpha + \beta)$$
----- (ii)

Combining equation (i) and (ii) gives

$$\theta = 2A.$$

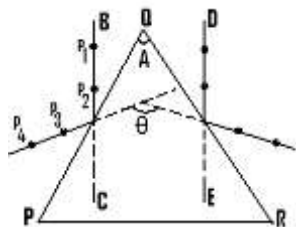
NOTE :

It is now clear from the geometry that the angle  $\theta$  turned through in moving the telescope from  $T_1$  to  $T_2$  is given by  $\theta = 2A$

Thus  $A = \frac{\theta}{2}$ .

## METHOD 2: USING OPTICAL PINS

A white paper is stuck to the soft board using top-headed pins. Two parallel lines AB and DC are drawn on the paper and the prism is placed with its apex as shown.



Two optical pins  $P_1$  and  $P_2$  are placed along AB and pins  $P_3$  and  $P_4$  are placed such that they appear to be in line with the images of  $P_1$  and  $P_2$  as seen by reflection from face PQ. The procedure is repeated for face QR. The prism is removed and angle  $\theta$  is measured. The required refracting angle  $A = \frac{\theta}{2}$

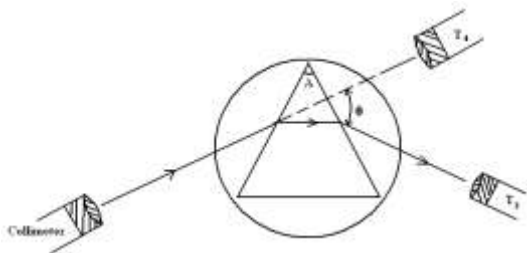
## MEASUREMENT OF MINIMUM DEVIATION " $D_{\min}$ " OF THE PRISM

The collimator is adjusted to produce parallel rays of light.

The turn table is leveled.

The telescope is adjusted to receive light from the collimator on its cross wire

The prism is placed on a turn table with its refracting angle facing away from the collimator as shown.



The telescope is turned in the direction of the base of the prism until light can be seen. With light kept in view, both the telescope and the table are turned until light moves in the opposite direction. Position  $T_3$  of the telescope is noted.

The table is then fixed and the prism is removed so that the telescope is turned to a new position  $T_4$  to receive the undeviated light. The angle between  $T_3$  and  $T_4$  is determined and this is the angle of minimum deviation  $D_{\min}$ .

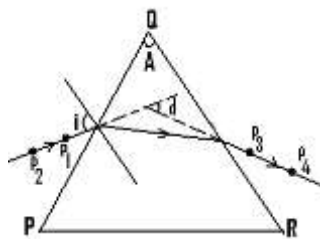
NOTE:

(i) Position  $T_3$  is noted because, in the position of minimum deviation light viewed through the telescope moves in the opposite direction.

(ii) The refracting index of a glass prism of known refracting angle  $A$  can be determined

$$\text{using a spectrometer from the relation } n = \frac{\sin \left( \frac{D_{\min} + A}{2} \right)}{\sin \frac{A}{2}}$$

## METHOD 2: USING OPTICAL PINS



Two optical pins  $P_1$  and  $P_2$  are placed along the lines that make different angles of incidence  $i$ . Pins  $P_3$  and  $P_4$  are placed such that they appear to be in line with the images of  $P_1$  and  $P_2$  as seen through the prism. The angles of deviation  $d$  are measured for different angles of incidence. A graph of  $d$  against  $i$  is plotted to give a curve whose angle of deviation at its turning point is the angle of minimum deviation  $D_{\min}$  of the prism.

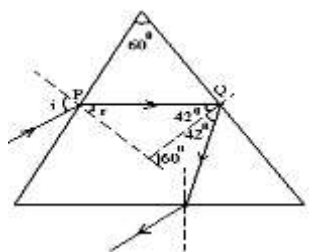
### USES OF A GLASS PRISM.

1. They enable the refractive index of a glass material to be measured accurately.
2. They are used in the dispersion of light emitted by glowing objects.
3. They are used as reflecting surfaces with minimal energy loss.
4. They are used in prism binoculars.

### More worked out examples

1. A ray of monochromatic light is incident on one face of a glass prism of refracting angle  $60^\circ$  and is totally internally reflected at the next face.

- (i) Draw a diagram to show the path of light through the prism.
- (ii) Calculate the angle of incidence at the first face of the prism if its refractive index is 1.53 and the angle of incidence at the second face is  $42^\circ$ .



From the diagram,  $r + 42^\circ = 60^\circ$   
 $\therefore r = 18^\circ$

At P, Snell's becomes

$$n_a \sin i = 1.53 \sin 18^\circ$$

$$\therefore i = 28.2^\circ$$

NOTE

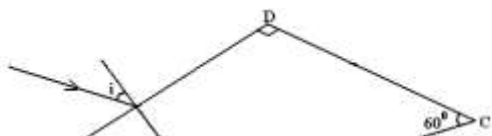
For  $n_g = 1.53$ , then the critical angle  $c$  for the above glass material is given by the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.53}$$

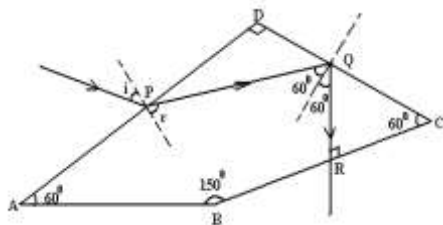
$$\therefore c = 40.8^\circ$$

Thus total internal reflection occurs at  $Q$  since the angle of incidence is greater than the critical angle  $c$

2. A ray of light is incident on the face AD of a glass block of refractive index 1.52 as shown.



If the ray emerges normally through face BC after total internal reflection, Calculate the angle of incidence, i.



#### ANALYSIS

(i) Its after a total internal reflection at Q that the ray emerges through face DC

(ii) At R, there is no refraction. There fore Snell's law does not hold at this point.

From  $\triangle RCQ$ ,  $\angle RQC + 60^\circ + 90^\circ = 180^\circ$

$$\therefore \angle RQC = 30^\circ$$

$\Rightarrow$  At Q, the angle of reflection =  $60^\circ$

Hence at Q, the angle of incidence =  $60^\circ$

Solving  $\triangle QDP$  gives  $\angle QPD = 60^\circ$

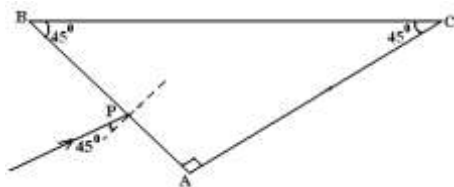
Hence at P, the angle of refraction  $r = 30^\circ$

$\Rightarrow$  At P, Snell's law becomes

$$n_a \sin i = 1.52 \sin 30^\circ$$

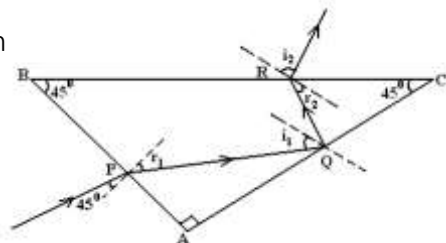
$$\therefore i = 49.5^\circ$$

3. A ray of light is incident at  $45^\circ$  on a glass prism of refractive index 1.5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

#### Solution



At P, Snell's becomes  $n_a \sin 45^\circ = 1.5 \sin r$

$$\therefore r_1 = 28.1^\circ$$

At P,  $\angle APQ + r_1 = 90^\circ$  where  $r_1 = 28.1^\circ$

$$\Rightarrow \angle APQ = 61.9^\circ$$

From  $\triangle APQ$ ,  $\angle PQA + 61.9^\circ + 90^\circ = 180^\circ$

$$\therefore \angle PQA = 28.1^\circ$$

$\Rightarrow$  At Q, the angle of incidence  $i = 61.9^\circ$

**Testing for total internal reflection at Q using the relation  $\sin c = \frac{1}{n_g} = \frac{1}{1.5}$  gives  $c = 41.8^\circ$**

Thus light is totally reflected at Q since  $i > c$ .

$$\Rightarrow \angle PQA = \angle RQC = 28.1^\circ$$

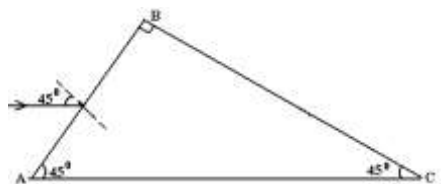
$$\text{From } \triangle RQC, 28.1^\circ + 45^\circ + 90^\circ + r_2 = 180^\circ$$

$$r_2 = 16.9^\circ$$

$$\text{At R, Snell's becomes } 1.5 \sin 16.9^\circ = n_a \sin i_2$$

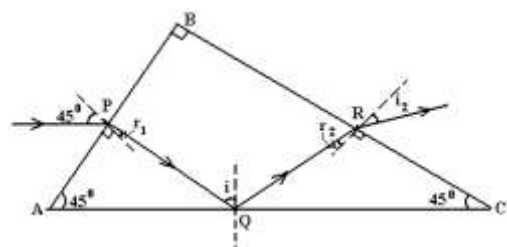
$$\text{Thus } i_2 = 25.15^\circ$$

4. A ray of light is incident at  $45^\circ$  on a glass prism of refractive index 1.5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

Solution



$$\text{At P, Snell's becomes } n_a \sin 45^\circ = 1.5 \sin r$$

$$\therefore r_1 = 28.1^\circ$$

$$\text{From } \triangle APQ, \angle PQA + 45^\circ + 90^\circ + r_1 = 180^\circ$$

$$\text{where } r_1 = 28.1^\circ$$

$$\therefore \angle PQA = 16.9^\circ$$

$$\Rightarrow \text{At Q, the angle of incidence } i = 73.1^\circ$$

$$\text{Testing for total internal reflection at Q using the relation } \sin c = \frac{1}{n_g} = \frac{1}{1.5} \text{ gives } c = 41.8^\circ$$

Thus light is totally reflected at Q since  $i > c$ .

$$\Rightarrow \angle PQA = \angle RQC = 16.9^\circ$$

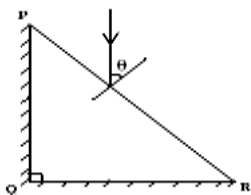
$$\text{From } \triangle RQC, 16.9^\circ + 45^\circ + 90^\circ + r_2 = 180^\circ$$

$$r_2 = 28.1^\circ$$

$$\text{At R, Snell's becomes } 1.5 \sin 28.1^\circ = n_a \sin i_2$$

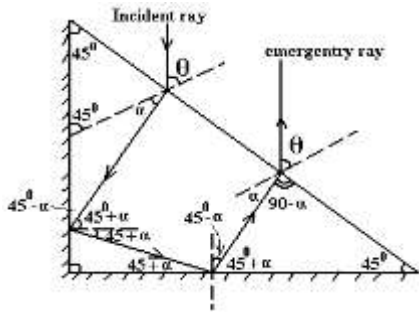
$$\text{Thus } i_2 = 45^\circ$$

5. The diagram in the figure below shows a cross section of an isosceles right angled prism sides PQ and QR are coated with a reflecting substance. A ray of light is incident on PR at an angle  $\theta$  as shown



(i) Draw a diagram to show the path of light through the prism.

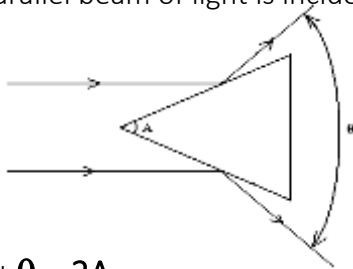
(ii) Show that the ray leaving the prism is parallel to the incident ray.



From geometry of the figure above the angle of emergence at D is the same as the incident angle  $\theta$  at A. Hence the emergent ray is parallel to the incident ray.

#### EXERCISE

1. Draw a labeled diagram of a spectrometer and State the necessary adjustments that must be made on to it before put in to use.
2. Describe how the refracting angle of the prism can be measured using a spectrometer.
3. You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the refracting angle  $A$  of the prism
  1. A parallel beam of light is incident on to a prism of refracting angle,  $A$ , as shown



Show that  $\theta = 2A$

5. Describe how the minimum deviation,  $D$ , of a ray of light passing through a glass prism can be measured using a spectrometer.
6. You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the angle of minimum deviation,  $D$ , of a ray of light passing through a glass prism.
7. Describe how the refractive index of a material of a glass prism of known refracting angle can be determined using a spectrometer.
8. Describe briefly two uses of glass prisms